Wire and Strip Conductors over a Dielectric-Coated Conducting or Dielectric Half-Space

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Abstract —**The complex wavenumber and characteristic impedance are determined for a wire or flat strip over a dielectric-coated half-space that may be a conductor or a dielectric with large permittivity. Elevated microstrip is an example of the configuration. The properties of the wire as an antenna or transmission line are determined from those of the insulated antenna with a two-layer eccentric insulation. The theory is extended to the strip conductor with the help of a comparison of the tubular and strip conductors over a perfectly conducting half-space.**

I. Introduction

THE PROPERTIES of the horizontal wire antenna over a conducting earth are well known [1]. They were derived from the theory of the dipole antenna with an eccentric dielectric coating [2]. This analysis was subsequently extended to the wave antenna of the generalized Beverage type [3]. The procedure carried out in this earlier study is applied in this paper to the horizontal wire antenna over the same half-space but when it is coated with a layer of dielectric material. In the formulation use is made of the analysis of the shielded transmission line with an eccentric inner conductor [4], [5]. As a final step, the properties of the strip antenna or transmission line are derived from those of the wire conductor with the help of the identity of the integral equations for coupled tubular and strip conductors [6].

The analysis resembles that of open-wire transmission lines in that radiation into the air is neglected in the determination of the wavenumber and characteristic impedance and, therefore, of the current distribution. This is an excellent approximation when the conducting wire or strip is located at small electrical distances from the dielectric-coated half-space. Once the current distribution is known in terms of its dependence on the electrical properties of the several media involved, the electromagnetic field can be calculated from the known field of a unit horizontal dipole over a three-layered region [7].

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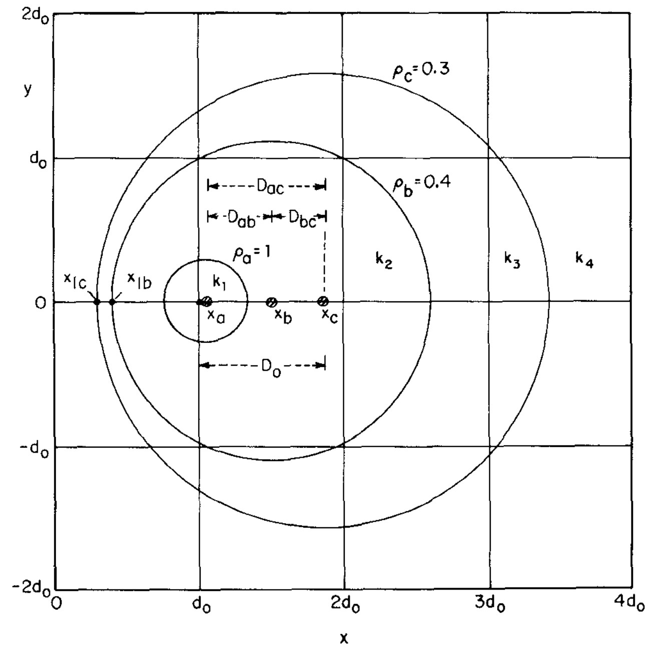

Fig. 1. Three equipotential circles with $\rho_c = 0.3$, $\rho_b = 0.6$, and $\rho_a = 1$. The wavenumbers of the four regions bounded by these circles are k_j, $j = 1,2,3,4$.

II. Review of the Theory of the Two-Layer Eccentrically Insulated Conductor

The cross section of the cylindrical structure to be studied initially is shown in Fig. 1. It consists of four regions separated by three circular boundaries with the radii a, b, and c. These boundaries are equipotential surfaces for the vector potential $\vec{A} = \hat{z}A_z$ and the scalar potential φ. The four regions are: region 1, a conducting wire or tube with radius a and complex wavenumber $k_1 = \beta_1 + i\alpha_1 = (1+i)(\omega\mu_0\sigma_1/2)^{1/2}$; region 2, air with the radius b and the wavenumber $k_2 = \omega(\mu_0\epsilon_0)^{1/2}$; region 3, a dielectric with radius c and the wavenumber $k_3 = \beta_3 + i\alpha_3 = \omega\mu_0^{1/2}(\epsilon_3 + i\sigma_3/\omega)^{1/2}$ with $\sigma_3/\omega\epsilon_3 \ll 1$; and region 4, the half-space outside the dielectric with the wavenumber $k_4 = \beta_4 + i\alpha_4 = \omega\mu_0^{1/2}(\epsilon_4 + i\sigma_4/\omega)^{1/2}$. It is required that

$$|k_4| \gg |k_3| > k_2. \tag{1}$$

It is assumed that in each cross-sectional plane the two-dimensional Laplace equation is an adequate approximation, but that the wave equation applies in the axial direction, z. Thus, for the vector potential, the equation in each transverse plane is

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = 0 \tag{2}$$

and the appropriate solution is

$$A_z = \frac{\mu_0 I_z(z)}{2\pi} \ln\left[\frac{(x+d_0)^2+y^2}{(x-d_0)^2+y^2}\right]^{1/2} = 2\rho K(z) \tag{3}$$

where d_0 is a constant distance and $K = \mu_0 I_z(z)/2\pi$ is a function of the axial coordinate z alone. $I_z(z)$ is the total axial current in the conductor with the radius a. The equipotential circles with constant ρ are defined by

$$(x+d_0)^2+y^2 = e^{4\rho}\left[(x-d_0)^2+y^2\right]$$

or

$$(x-d_0\coth 2\rho)^2+y^2 = \frac{d_0^2}{\sinh^2 2\rho} \tag{4}$$

where $0 \leqslant \rho \leqslant \infty$. Note that the surface $\rho = 0$ is the yz plane, $x = 0$; and $\rho = \infty$ defines the line $x = d_0$, $y = 0$.

The center of a typical circle is at

$$x = d_0 \coth 2\rho \qquad y = 0 \tag{5}$$

and its radius is

$$r = \frac{d_0}{\sinh 2\rho}. \tag{6}$$

The three circles shown in Fig. 1 have their centers at

$$x_a = d_0\coth 2\rho_a \qquad x_b = d_0 \coth 2\rho_b \qquad x_c = d_0\coth 2\rho_c \tag{7}$$

and the radii

$$a = \frac{d_0}{\sinh 2\rho_a} \qquad b = \frac{d_0}{\sinh 2\rho_b} \qquad c = \frac{d_0}{\sinh 2\rho_c}. \tag{8}$$

Note that the circle of zero radius and $\rho = \infty$ is at

$$x_0 = d_0 = a\sinh 2\rho_a = b \sinh 2\rho_b = c\sinh 2\rho_c. \tag{9}$$

The distance d_0 is a principal parameter.

The points of intersection of the circles with the x axis are at

$$x_{1_j} = d_0 \tanh \rho_j \quad \text{and} \quad x_{2_j} = d_0 \coth \rho_j, \qquad j = a, b, c. \tag{10}$$

The shortest distance between the circle ρ_b and ρ_c is

$$l \equiv x_{1b} - x_{1c} = d_0(\tanh\rho_b - \tanh\rho_c). \tag{11}$$

It is a second principal parameter.

It is convenient to refer the potentials on the several circles to the potential on circle c and define the following potential differences:

$$W_{az} = A_{az} - A_{cz} = 2K(\rho_a - \rho_c) \tag{12}$$

$$W_{bz} = A_{bz} - A_{cz} = 2K(\rho_b - \rho_c). \tag{13}$$

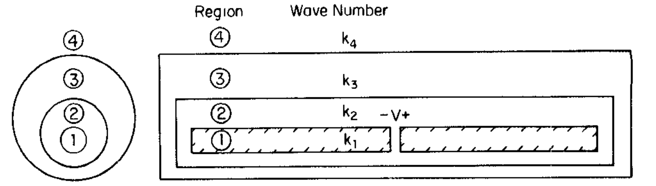

Fig. 2. Center-driven two-layer (regions 2 and 3) eccentrically insulated conductor (region 1) in an infinite dielectric or conducting region 4.

The equipotential circles can be expressed in terms of their radii and the distances D between their centers. These latter are

$$D_{ac} \equiv x_c - x_a \qquad D_{bc} \equiv x_c - x_b$$
$$D_{ab} \equiv x_b - x_a \qquad D_0 = x_c - d_0. \tag{14}$$

With (7) and (8),

$$2\rho_a = \cosh^{-1}\left(\frac{c^2-a^2-D_{ac}^2}{2aD_{ac}}\right) = \cosh^{-1}\left(\frac{b^2-a^2-D_{ab}^2}{2aD_{ab}}\right) \tag{15a}$$

$$2\rho_b = \cosh^{-1}\left(\frac{c^2-b^2-D_{bc}^2}{2bD_{bc}}\right) = \cosh^{-1}\left(\frac{b^2-a^2+D_{ab}^2}{2bD_{ab}}\right) \tag{15b}$$

$$2\rho_c = \cosh^{-1}\left(\frac{c^2-a^2+D_{ac}^2}{2cD_{ac}}\right) = \cosh^{-1}\left(\frac{c^2-b^2+D_{bc}^2}{2cD_{bc}}\right). \tag{15c}$$

The normalized potential differences $\Omega = W_z/K$ are obtained with the formula

$$\cosh^{-1}x - \cosh^{-1}y = \cosh^{-1}\left[xy - \sqrt{(x^2-1)(y^2-1)}\right]. \tag{16}$$

Thus,

$$\Omega_{ac} = 2(\rho_a-\rho_c) = \cosh^{-1}\left(\frac{a^2+c^2-D_{ac}^2}{2ac}\right) \tag{17a}$$

$$\Omega_{bc} = 2(\rho_b-\rho_c) = \cosh^{-1}\left(\frac{b^2+c^2-D_{bc}^2}{2bc}\right) \tag{17b}$$

$$\Omega_{ab} = 2(\rho_a-\rho_b) = \cosh^{-1}\left(\frac{a^2+b^2-D_{ab}^2}{2ab}\right). \tag{17c}$$

III. Transmission-Line Properties of the Two-Layer Eccentrically Insulated Conductor

The eccentrically insulated center-driven conductor is shown in Fig. 2. The wavenumber k_L for the current in the conductor and the characteristic impedance Z_c are given in

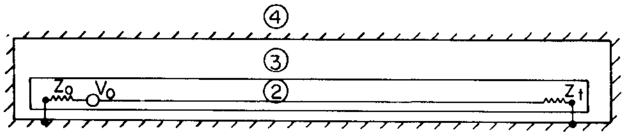

Fig. 3. Two-layer eccentrically insulated conductor in an infinite conducting region as a transmission line.

[5]. With minor changes in the notation, they are

$$k_L = k_2 F \left[1 + \frac{\Delta(D_{ac},0)}{k_4 c \Omega_{ac}}\right]^{1/2} \tag{18}$$

$$Z_c = \frac{k_L \zeta_2}{2\pi k_2}\left[\Omega_{ab} + (k_2/k_3)^2 \Omega_{bc}\right] \tag{19}$$

where $\zeta_2 = \zeta_0 = 120\pi\ \Omega$,

$$F = \left[\frac{\Omega_{ac}}{\Omega_{ab} + (k_2/k_3)^2 \Omega_{bc}}\right]^{1/2} \tag{20}$$

and

$$\Delta(D_{ac},0) = \frac{H_0^{(1)}(k_4 c)}{H_1^{(1)}(k_4 c)} + 2\sum_{m=1}^{\infty}\left(\frac{D_{ac}D_0}{c^2}\right)^m \frac{H_m^{(1)}(k_4 c)}{H_{m+1}^{(1)}(k_4 c)}. \tag{21}$$

The current in the center-driven conductor when it has the length $2h$ is

$$I_z(z) = -\frac{iV_0}{2Z_c}\frac{\sin k_L(h-|z|)}{\cos k_L h}. \tag{22}$$

When the conductor is infinitely long or is terminated in its characteristic impedance as shown in Fig. 3, the current is

$$I_z(z) = \frac{V_0}{2Z_c}e^{ik_L z}, \qquad z \geqslant 0. \tag{23}$$

IV. Theory of the Two-Layer Eccentrically Insulated Conductor over a Conducting or Dielectric Half-Space

The next step in the analysis is to let the radius c of the outer circular boundary between regions 3 and 4 become infinite ($c \to \infty$) so that $\rho_a \to 0$ and region 4 becomes a half-space with the plane boundary at $x = 0$, as shown in Fig. 4. If the nearest distance to the circle ρ_b is maintained at $x_{1b} - x_{1c} = l$, it follows from (10) that

$$\rho_b = \tanh^{-1}(l/d_0) \tag{24}$$

since $x_{1c} = 0$. With (8) the radius b is

$$b = \frac{d_0}{\sinh\left[2\tanh^{-1}(l/d_0)\right]} \tag{25}$$

and, with (7), the center of the circle is at

$$x_b = d_0 \coth\left[2\tanh^{-1}(l/d_0)\right]. \tag{26}$$

In order to take the limit $c \to \infty$, $x_c \to \infty$, it is necessary to obtain an explicit formula for b in terms of the distance l, the radius c, and the distance $D_0 = x_c - d_0$. With (7) and

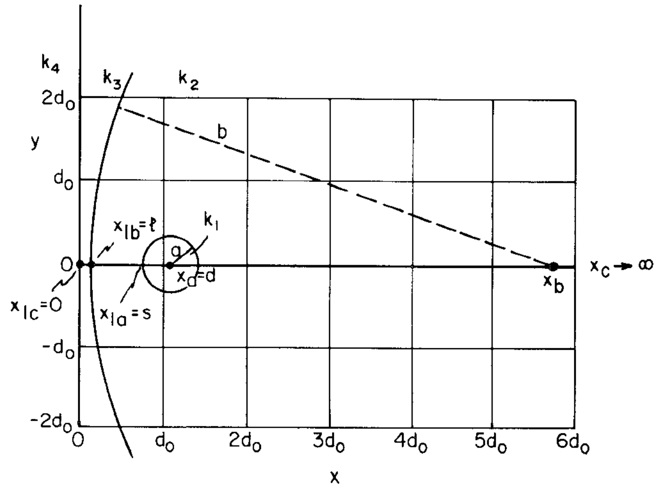

Fig. 4. Three equipotential circles with $\rho_c = 0$, $\rho_b = 0.088$, and $\rho_a = 1$

(8), it follows that $(x_c/c)^2 - (d_0/c)^2 = 1$ so that

$$d_0 = \sqrt{x_c^2 - c^2} \quad \text{or} \quad x_c = \sqrt{d_0^2 + c^2} \tag{27a}$$

$$D_0 = x_c - \sqrt{x_c^2 - c^2} \quad \text{or} \quad x_c = (D_0^2 + c^2)/2D_0. \tag{27b}$$

It follows from (7) and (8) with (15b), viz.,

$$\frac{x_b}{b} = \cosh 2\rho_b = \frac{c^2 - b^2 - D_{bc}^2}{2bD_{bc}} \tag{28}$$

that

$$b^2 + 2x_b D_{bc} - c^2 + D_{bc}^2 = 0. \tag{29}$$

With $x_b = x_c - D_{bc}$ and $D_{bc} = c - b - l$,

$$b = \frac{2(c-l)(x_c - c) - l^2}{2(x_c - c + l)}. \tag{30}$$

With (27a),

$$b = \frac{2(c-l)\left(\sqrt{c^2 + d_0^2} - c\right) - l^2}{2\left(\sqrt{c^2 + d_0^2} - c + l\right)}. \tag{31}$$

It is now possible to let $c \to \infty$, $x_c \to \infty$. With $\sqrt{c^2 + d_0^2} \to c + (d_0^2/2c)$, it follows that $2(c-l)(\sqrt{c^2 + d_0^2} - c) \to 2c(d_0^2/2c) = d_0^2$ and $\sqrt{c^2 + d_0^2} - c \to 0$, so that

$$b = \frac{d_0^2 - l^2}{2l}. \tag{32}$$

The center of the circle ρ_b is at

$$x_b = b + l = \frac{d_0^2 + l^2}{2l}. \tag{33}$$

The corresponding formulas for the circle with the radius a are obtained from

$$x_b - x_a = b\cosh 2\rho_b - a\cosh 2\rho_a = \frac{c^2 - b^2 - D_{bc}^2}{2D_{bc}} - \frac{c^2 - a^2 - D_{ac}^2}{2D_{ac}} \tag{34}$$

where

$$D_{ac} = x_c - x_a \quad \text{and} \quad D_{bc} = x_c - x_b. \tag{35}$$

When (35) is used in (34), this can be rearranged to give

$$(x_b - x_a)[(x_c - x_a)(x_c - x_b) - c^2 - 1] + b^2(x_c - x_a) - a^2(x_c - x_b) = 0. \tag{36}$$

When $c \to \infty$, $x_c \to \infty$, this yields

$$x_b^2 - x_a^2 = b^2 - a^2. \tag{37}$$

With (32) and (33),

$$x_a^2 = x_b^2 - b^2 + a^2 = \frac{(d_0^2 + l^2)^2}{4l^2} - \frac{(d_0^2 - l^2)^2}{4l^2} + a^2 = d_0^2 + a^2 \tag{38}$$

or

$$x_a \equiv d = \sqrt{d_0^2 + a^2} \quad \text{and} \quad d_0 = \sqrt{d^2 - a^2}. \tag{39}$$

It is convenient to denote the distance x_a from the boundary $x = 0$ to the center of the conductor with the radius a by d since this is a principal parameter.

It remains to evaluate the functions Ω defined in (17a)–(17c) in the limit $c \to \infty$, $x_c = \sqrt{c^2 + d_0^2} \to \infty$. The results are

$$\Omega_{ac} = \cosh^{-1}\left(\frac{a^2 + c^2 - D_{ac}^2}{2ac}\right) = \cosh^{-1}\left[\frac{a^2 + c^2 - (x_c - x_a)^2}{2ac}\right] \to \cosh^{-1}\frac{x_a}{a} = \cosh^{-1}\frac{d}{a} \tag{40a}$$

$$\Omega_{bc} = \cosh^{-1}\left(\frac{b^2 + c^2 - D_{bc}^2}{2bc}\right) = \cosh^{-1}\frac{x_b}{b} = \cosh^{-1}\left(\frac{d_0^2 + l^2}{d_0^2 - l^2}\right) = \cosh^{-1}\left(\frac{d^2 - a^2 + l^2}{d^2 - a^2 - l^2}\right) \tag{40b}$$

$$\Omega_{ab} = \cosh^{-1}\left(\frac{a^2 + b^2 - D_{ab}^2}{2ab}\right) = \cosh^{-1}\left[\frac{a^2 + b^2 - (x_b - x_a)^2}{2ab}\right]$$

so that, with (32), (33) and (39),

$$\Omega_{ab} = \cosh^{-1}\left[\frac{d(d^2 - a^2 + l^2) - 2l(d^2 - a^2)}{a(d^2 - a^2 - l^2)}\right]. \tag{40c}$$

These quantities are expressed in terms of the thickness l of the dielectric layer, the radius a of the conductor, and the height d of its center over the plane boundary $x = 0$ between the dielectric layer and the conducting or dielectric half-space.

As a check on (40a)–(40c), use can be made of (20), viz.,

$$F^2 = \frac{\Omega_{ac}}{\Omega_{ab} + (k_2/k_3)^2\Omega_{bc}}. \tag{41}$$

It is readily verified with the formula $\cosh^{-1}x + \cosh^{-1}y = \cosh^{-1}[xy + \sqrt{(x^2-1)(y^2-1)}]$ that, when $k_2 = k_3$, $F^2 = 1$.

It remains to evaluate $\Delta(D_{ac}, 0)$ in (21) in the limit $c \to \infty$, $x_c \to \infty$. The first step is to determine

$$\frac{D_{ac}D_0}{c^2} = \frac{(x_c - x_a)(x_c - d_0)}{c^2} = \frac{\left(\sqrt{d_0^2 + c^2} - \sqrt{d_0^2 + a^2}\right)\left(\sqrt{d_0^2 + c^2} - d_0\right)}{c^2} \to \left(1 - \frac{\sqrt{d_0^2 + a^2}}{c}\right)\left(1 - \frac{d_0}{c}\right). \tag{42}$$

With (42), (21) becomes

$$\Delta(D_{ac}, 0) = \frac{H_0^{(1)}(k_4c)}{H_1^{(1)}(k_4c)} + 2\sum_{m=1}^{\infty}\left(1 - \frac{d}{c}\right)^m \cdot \left(1 - \frac{\sqrt{d^2 - a^2}}{c}\right)^m \frac{H_m^{(1)}(k_4c)}{H_{m+1}^{(1)}(k_4c)}. \tag{43}$$

It is shown in [1] that, in the limit $c \to \infty$,

$$\frac{\Delta(D_{ac}, 0)}{k_4c} \equiv \Delta(A) = 2\left[\frac{1}{A^2} - \frac{K_1(A)}{A} + \frac{i\pi I_1(A)}{2A} - i\left(\frac{A}{3} + \frac{A^3}{45} + \frac{A^5}{1575} + \cdots\right)\right]^{1/2}, \tag{44}$$

where K_1 and I_1 are the modified Bessel functions, and

$$A = k_4(d + \sqrt{d^2 - a^2}). \tag{45}$$

When (44), (41) and (40a)–(40c) are substituted in (18) and (19), these give

$$k_L = k_2\left[\frac{\Omega_{ac} + \Delta(A)}{\Omega_{ab} + (k_2/k_3)^2\Omega_{bc}}\right]^{1/2} \tag{46}$$

$$Z_c = \frac{\zeta_2}{2\pi}\left[\Omega_{ab} + (k_2/k_3)^2\Omega_{bc}\right]^{1/2}\left[\Omega_{ac} + \Delta(A)\right]^{1/2}. \tag{47}$$

V. The Series Impedance and Shunt Admittance per Unit Length

The complex wavenumber and characteristic impedance can be expressed in terms of the series impedance per unit length z_L and the shunt admittance per unit length y_L. These are related to k_L and Z_c by

$$k_L = \sqrt{-z_L y_L} \qquad Z_c = \sqrt{z_L / y_L} \tag{48}$$

or

$$z_L = -ik_L Z_c = z^e + z_4^i \tag{49}$$

$$\frac{1}{y_L} = \frac{Z_c}{ik_L} = \frac{1}{y_2} + \frac{1}{y_3}. \tag{50}$$

With (46) and (47),

$$z_L = z^e + z_4^i = -\frac{ik_L^2 \zeta_2}{2\pi k_2}\left[\Omega_{ab} + (k_2/k_3)^2 \Omega_{bc}\right]$$

$$= -\frac{i\omega\mu_0}{2\pi}\left[\Omega_{ac} + \Delta(A)\right] \tag{51}$$

so that the external impedance per unit length z^e and the internal impedance per unit length z_4^i of the conducting half-space are

$$z^e = -i\omega l^e = -\frac{i\omega\mu_0}{2\pi}\Omega_{ac} \tag{52}$$

$$z_4^i = -\frac{i\omega\mu_0}{2\pi}\Delta(A)$$

$$= -\frac{i\omega\mu_0}{\pi}\left[\frac{1}{A^2} - \frac{K_1(A)}{A} + \frac{i\pi I_1(A)}{2A} - i\left(\frac{A}{3} + \frac{A^3}{45} + \frac{A^5}{1575} + \cdots\right)\right]^{1/2} \tag{53a}$$

with

$$A = k_4\left(d + \sqrt{d^2 - a^2}\right). \tag{53b}$$

The formula (51) does not include the internal impedance of the conductor. This can be added to (51) so that

$$z_L = z_1^i + z_4^i + z^e \tag{54}$$

where

$$z_1^i = -\frac{ik_1}{2\pi a \sigma_1} = \frac{1-i}{2\pi}\left(\frac{\omega\mu_0}{2\sigma_1}\right)^{1/2}. \tag{55}$$

With (46) and (47), (50) gives

$$\frac{1}{y_L} = \frac{i\zeta_2}{2\pi k_2}\left[\Omega_{ab} + (k_2/k_3)^2\Omega_{bc}\right] = \frac{1}{y_2} + \frac{1}{y_3} \tag{56}$$

so that

$$\frac{1}{y_2} = \frac{i\zeta_2}{2\pi k_2}\Omega_{ab} \qquad \frac{1}{y_3} = \frac{i\zeta_2}{2\pi k_3}\Omega_{bc}. \tag{57}$$

With $\zeta_2/k_2 = 1/\omega\epsilon_0$, $\zeta_2 k_2/k_3^2 = 1/(\omega\epsilon_3 + i\sigma_3)$,

$$y_2 = -i\omega c_2 = -\frac{2\pi i\omega\epsilon_0}{\Omega_{ab}}$$

$$y_3 = g_3 - i\omega c_3 = \frac{2\pi\sigma_3}{\Omega_{bc}} - \frac{2\pi i\omega\epsilon_3}{\Omega_{bc}}. \tag{58}$$

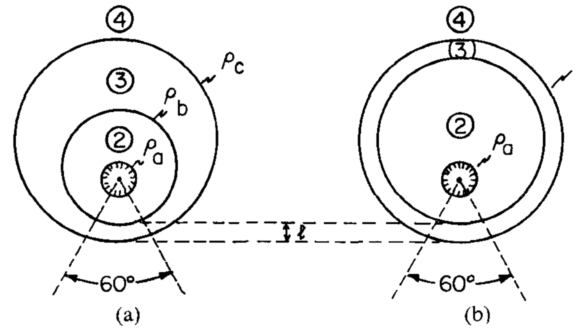

Fig. 5. Approximately equivalent two-layer eccentrically insulated conductors.

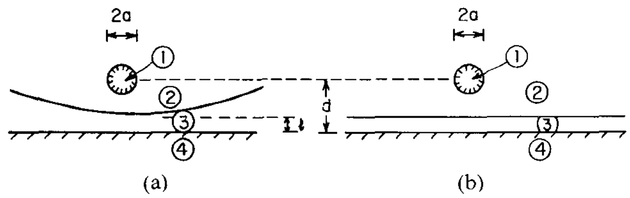

Fig. 6. Approximately equivalent two-layer insulated conductors when the radius of the boundary between regions 3 and 4 is infinite.

The final complete formulas for k_L and Z_c are

$$k_L = k_2\left[\frac{\Omega_{ac} + \dfrac{k_1}{\omega\mu_0 a\sigma_1} + \Delta(A)}{\Omega_{ab} + (k_2/k_3)^2\Omega_{bc}}\right]^{1/2} \tag{59}$$

$$Z_c = \frac{\zeta_2}{2\pi}\left[\Omega_{ab} + (k_2/k_3)^2\Omega_{bc}\right]^{1/2} \cdot \left[\Omega_{ac} + \frac{k_1}{\omega\mu_0 a\sigma_1} + \Delta(A)\right]^{1/2}. \tag{60}$$

When regions 1 and 4 are perfect conductors and regions 2 and 3 are perfect dielectrics,

$$k_L = k_2\left[\frac{\Omega_{ac}}{\Omega_{ab} + (\epsilon_2/\epsilon_3)\Omega_{bc}}\right]^{1/2} \tag{61}$$

$$Z_c = \frac{\zeta_2}{2\pi}\left\{\Omega_{ac}\left[\Omega_{ab} + (\epsilon_2/\epsilon_3)\Omega_{bc}\right]\right\}^{1/2}. \tag{62}$$

When $\epsilon_3 = \epsilon_2$,

$$k_L = k_2 \qquad Z_c = \frac{\zeta_2}{2\pi}\Omega_{ac}. \tag{63}$$

These are the values for a perfectly conducting wire over a perfectly conducting half-space.

VI. The Eccentrically Insulated Tubular Conductor over a Dielectric-Coated Dielectric or Conducting Half-Space

The formulas for k_L and Z_c derived in Sections IV and V apply to the structure shown in Fig. 5(a) when the radius c of the boundary between regions 3 and 4 is made infinite to obtain the configuration in Fig. 6(a). Region 4 is now a half-space with a plane boundary between it and the dielectric layer which has a large radius b and a minimum thickness l.

In practice, the configuration shown in Fig. 6(b) is more useful. It consists of the conductor (region 1) with radius a in air (region 2) over a half-space (region 4) that is coated with a dielectric (region 3) with the uniform thickness l. This can be looked upon as the eccentrically insulated conductor shown in Fig. 5(b) in the limit $c \to \infty$, $b \to \infty$.

It is shown in [5] that the electromagnetic field on the circle ρ_b in Fig. 5(a) decreases rapidly outside the 60° angle when $a/c = 0.1$ and $d/c = 1/3$, so that the field in the remaining 300° is very small and it is immaterial whether the dielectric in region 3 has the shape shown in Fig. 5(a) or Fig. 5(b) insofar as the values of k_L and Z_c are concerned. It follows that (18) and (19) should be good approximations for the configuration of Fig. 5(b) and, in the limit when $c \to \infty$, (59) and (60) for Fig. 6(a) should be good approximations for Fig. 6(b). Note, however, that with l and b fixed, $d - l$ cannot be made arbitrarily small without simultaneously letting the radius a approach the large radius b which goes with a small value of l.

VII. The Strip Conductor over a Dielectric-Coated Half-Space

The formulas (59)–(63) for the circular conductor over a dielectric-coated half-space can be modified to apply to a flat strip conductor (Fig. 7) with the width $2w = 4a$ and the thickness t. This latter is assumed to be small compared with the skin depth, so that the internal impedance per unit length of the strip is

$$z_1^i = \frac{1}{2w\sigma_c t} = \frac{1}{4a\sigma_c t}. \qquad (64)$$

It is shown in [6] that the constants z_L and y_L of the stripline with width $2w = 4a$ differ from those of the line with circular tubular conductors given in (51) and (56) only in the substitution of $\sqrt{d^2 + a^2}$ for d. It follows that, with (64) and (51),

$$z_L = z^e + z_1^i + z_4^i \qquad (65)$$

where

$$z^e = -i\omega l^e = -\frac{i\omega\mu_0}{2\pi}\Omega_{ac}$$

$$= -\frac{i\omega\mu_0}{2\pi}\cosh^{-1}\frac{\sqrt{d^2 + a^2}}{a} \qquad (66)$$

$$z_1^i = -\frac{i\omega\mu_0}{2\pi}\,\frac{i\pi}{2\omega\mu_0 a\sigma_c t} \qquad (67)$$

$$z_4^i = -\frac{i\omega\mu_0}{2\pi}\Delta(A) \qquad (68)$$

where

$$\Delta(A) \equiv 2\left[\frac{1}{A^2} - \frac{K_1(A)}{A} + \frac{i\pi I_1(A)}{2A} - i\left(\frac{A}{3} + \frac{A^3}{45} + \frac{A^5}{1575} + \cdots\right)\right]^{1/2} \qquad (69)$$

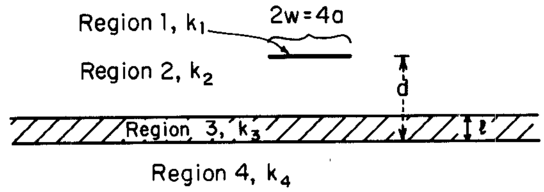

Fig. 7. Strip conductor in air over a dielectric-coated half-space.

and

$$A = k_4\left(d + \sqrt{d^2 + a^2}\right). \qquad (70)$$

Similarly,

$$\frac{1}{y_L} = \frac{1}{y_2} + \frac{1}{y_3} \qquad (71)$$

where

$$\frac{1}{y_2} = \frac{i\zeta_2}{2\pi k_2}\Omega_{ab}$$

$$= \frac{i\zeta_2}{2\pi k_2}\cosh^{-1}\left[\frac{\sqrt{d^2 + a^2}\,(d^2 + l^2) - 2ld^2}{a(d^2 - l^2)}\right] \qquad (72)$$

$$\frac{1}{y_3} = \frac{i\zeta_2}{2\pi k_3}\Omega_{bc}$$

$$= \frac{i\zeta_2}{2\pi k_3}\cosh^{-1}\left(\frac{d^2 + l^2}{d^2 - l^2}\right). \qquad (73)$$

As in general

$$Z_c = \sqrt{z_L/y_L}, \qquad k_L = \sqrt{-z_L y_L}. \qquad (74)$$

Throughout this section, $a = w/2$, where w is the half-width of the strip. The above formulas are accurate when

$$d \geqslant a = w/2. \qquad (75)$$

At $d = a$, the error in Z_c is 8 percent; at $d = 4a$, the error in Z_c is 0.7 percent.

VIII. Applications and Conclusion

The characteristic impedance and the complex wavenumber together with the associated series impedance and shunt admittance per unit length have been derived for a horizontal wire with radius a or strip with width $2w = 4a$ over a two-layer region. This consists of a layer of dielectric with thickness l over a conducting or dielectric half-space. The formulas can take account of losses in any or all of the four regions.

Possible applications of the results include long wave antennas [3] erected (a) on concrete or asphalt slabs over the earth or (b) over swamps, shallow ponds, lakes, or tidal basins. The theory also applies to horizontal dipoles over any two-layer region. In the determination of the wavenumber k_L and characteristic impedance Z_c for the current, radiation into the surrounding air is assumed to be small compared to the power transferred along the conductors and along the boundary as a lateral wave or in the dielectric layer as a surface wave. The complete field generated by the known currents in the conductors can be calculated from the formulas for the elementary horizontal dipole over a two-layer region [7].

The new theory of the strip transmission line applies to the elevated microstrip [8, p. 90, fig. 1.45(b)]; it does not apply directly to open microstrip transmission lines [8, p. 89, fig. 1.42(a)] because the conducting strip is located above and not on the dielectric substrate. It is nevertheless closely related to the quasi-TEM approximations used in open microstrip theory [8, pp. 96–99]. This depends on the empirical definition of an effective permittivity for the substrate [9]. When condition (75) is satisfied (instead of $d = 0$ for microstrip), the TEM mode is an accurate description of the propagation. As in microstrip, two dielectrics—air and the substrate with thickness l—are involved and an effective single permittivity is readily defined analytically. With (61) it is

$$\epsilon_e^{-1} = \frac{\epsilon_2^{-1}\Omega_{ab} + \epsilon_3^{-1}\Omega_{bc}}{\Omega_{ac}}. \tag{76}$$

In addition, the new theory includes the losses in the conducting strip, the dielectric substrate, and the conducting ground plane. Since they are due primarily to skin effect, they are frequency-dependent and this frequency dependence is properly contained in the formulas for the wavenumber and the characteristic impedance. It is significant, in this connection, that the losses in microstrip at frequencies above 40 GHz are primarily due to skin effect in the two conductors.

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